# _LITfinalLOGO

# SUMMER EXAMINATIONS 2013

**Monday, 13th May 2013, 14.30 p.m. – 17.00 p.m.**

**KSDEM\_8\_Y2**

**Programme:** Bachelor of Science (Honours) in Software Development

**Stage:** Two

**Module:** Mathematics for Computing

**Time Allowed:** 2½ Hours

**Instructions:**

**1.** Answer **FOUR** of the following **FIVE** questions.

**2.** All questions are equally weighted.

**Additional Attachments or Exam Material to accompany this paper:**

1. None

**Internal Examiner(s): External Examiners(s):**

Dr Oliver Hyde Mr. Brian Gillespie

**Q. 1**

**(a)**

Consider the set of the ten most popular Rihanna songs, taken from a recent poll, with elements {Umbrella, Disturbia, Only Girl (In the World), Don’t Stop the Music, What’s My Name, Take a Bow, Rude Boy, Shut Up and Drive, Unfaithful, We Found Love}.

1. How many subsets does this set have?
2. List the elements of the 682nd subset.
3. List the elements of the 438th subset.

**(6 marks)**

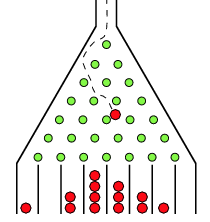
**(b)**

A group of nineteen people were discussing whether they liked or disliked four popular operating systems: Windows XP, Windows 7, Mac OS X, and Ubuntu. Seven members of the group dislike Windows XP, ten dislike Windows 7, eleven dislike Mac OS X, and six dislike Ubuntu. Five of the group dislike both Windows XP and Windows 7, five dislike both Windows XP and Mac OS X, six dislike Windows XP and Ubuntu, three dislike Windows 7 and Mac OS X, four dislike Windows 7 and Ubuntu, and five dislike Mac OS X and Ubuntu. Three people dislike Windows XP, Windows 7, and Mac OS X, while two dislike Windows XP, Windows 7 and Ubuntu; three dislike Windows XP, Mac OS X and Ubuntu; and four dislike Windows 7, Mac OS X and Ubuntu. Two people dislike all four operating systems. How many members of the group like all four operating systems?

**(7 marks)**

**(c)**

The *Galton Board*, or bean machine, is a device for statistical experiments named after English scientist Sir Francis Galton. It consists of an upright board with evenly spaced pegs driven into its upper half, where the pegs are arranged in staggered order, and a lower half divided into a number of evenly-spaced rectangular bins. The front of the device is covered with a glass cover to allow viewing of both pegs and bins. In the middle of the upper edge, there is a funnel into which balls can be dropped, where the diameter of the balls must be much smaller than the distance between the pegs. The funnel is located precisely above the central peg of the second row so that each ball, if perfectly centred, would fall vertically and directly onto the uppermost point of this peg’s surface. The figure below shows a Galton Board with seven rows of pegs. Each time a ball hits one of the pegs, it can bounce right (or left) with probability ½.



Pegs

Bins

1. Find the number of paths to each bin.
2. A ball is dropped into the Galton Board. What is the probability that:
   * It lands in the bin on the right?
   * It lands in the third bin from the left? **(7 marks)**

**(d)**

According to a puzzle book published in the Middle Ages, the statue of the goddess Pallas Athene was inscribed with the following information:

‘I, Pallas, am made from the purest gold, donated by five generous poets. Kariseus gave half; Thespian an eighth. Solon gave one-tenth; Themison gave one-twentieth. And the remaining nine talents’ worth of gold was provided by the good Aristodokos.’

What is the total cost of the statue, in talents[[1]](#footnote-1)? **(5 marks)**

**(Total 25 Marks)**

**Q. 2**

**(a)**

An International Standard Book Number (ISBN-10) is a code of 10 characters ignoring dashes, such as 1409144992 (“Triggs: The Autobiography of Roy Keane’s Dog” by Triggs). An ISBN-10 code consists of four parts: a group code, a publisher code, a code that uniquely identifies the book among those published by the particular publisher, and lastly a check character. The check character is used to validate an ISBN-10.

The check character is *s* mod 11, where *s* is the sum of the first digit plus two times the second digit plus three times the third digit, … plus nine times the ninth digit. If this value is 10, the check character is *X*.

Find the character ***C*** so that each of the following numbers is a valid ISBN-10 code:

1. 147112754***C*** – “Seven Deadly Sins” by David Walsh.

**(3 marks)**

1. 14091416***C***2 – “A Week in Winter” by Maeve Binchy.

**(4 marks)**

**(b)**

Find the greatest common divisor of these pairs of integers:

1. 4321 and 789;

**(3 marks)**

1. 3368 and 2848.

**(3 marks)**

**(c)**

Let *n* be any integer. What is (4*n*2 – 12*n* + 3) mod 4?

**(3 marks)**

**(d)**

The Division Algorithm states that for every two integers *a* and *b* with *b* > 0 there exist unique integers *q* and *r* such that *a = bq* + *r* where 0 ≤ r < |b|. Find the quotient *q* and remainder *r* as defined by the Division Algorithm for each of the following cases:

1. *a* = 14,206; *b* = –37;

**(3 marks)**

1. *a* = –3,456; *b* = –421.

**(3 marks)**

**(e)**

Provide a counterexample to the following statement about integers that is false:

If *a* mod *c* = *b* mod *c* then *a* = *b*.

**(3 marks)**

**(Total 25 Marks)**

**Q. 3**

**(a)**

The Monster Raving Fine Fáil Party has promised to build an underground rail system in Limerick if elected to power in the next election. They plan to have three circular lines, as shown in the following table.

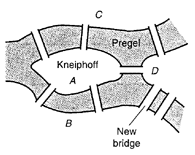
|  |  |  |
| --- | --- | --- |
| Richard Harris  Line | Kate O’Brien  Line | Frank McCourt  Line |
| Henry Street | Castletroy | Henry Street |
| Ennis Road | Parkway | William Street |
| Caherdavin | Garryowen | Colbert Station |
| Moylish | William Street | Punch’s Cross |
| Ballynanty | The Island | Dooradoyle |
| Treaty Stone | Corbally | Raheen |
| Charlotte’s Quay | Rhebogue | Dock Road |
| Henry Street | Castletroy | Henry Street |

Draw a graph showing all lines and stops for the proposed underground rail system.

**(10 marks)**

**(b)**

Consider the bridges of Königsberg problem, with a new bridge connecting land mass *B* to island *D,* as shown below.

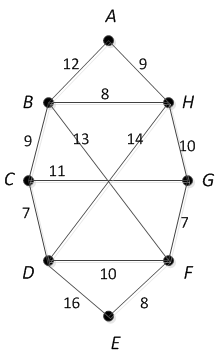


1. Is it possible to tour the region starting and finishing in the same area, having walked over every bridge exactly once? Either describe such a tour or explain why none is possible.
2. Is it possible to tour the region (with different starting and finishing points), having walked over every bridge exactly once? Either describe such a tour or explain why none is possible.

**(8 marks)**

**(c)**

Find a minimum spanning tree for the following graph. What is the smallest length of pavement required to connect the towns in this graph?



**(7 marks)**

**(Total 25 Marks)**

**Q. 4**

**(a)**

Give English translations of the following predicate formulae if

*L(x, y)* is “*x* loves *y*”.

*H(x)* is “*x* is handsome”.

*M(x)* is “*x* is a man”.

*P(x)* is “*x* is pretty”.

*W(x)* is “*x* is a woman”.

*r* is “Richard”.

*j* is “Julia”.

1. *P(j)* ∧ *L(r, j)*

**(2 marks)**

1. ∃*x* [*M(x)* ∧ *H(x*) ∧ *L(x, j*)]

**(3 marks)**

1. ∀*x* [*W(x)* ∧ *P(x*) → *L(r, x)*]

**(3 marks)**

**(b)**

You are the ruler of a medieval empire and you are about to have a celebration tomorrow. The celebration is the most important occasion you have ever hosted. You have 1,000 barrels of wine that you were planning to open for the celebration, but you find out that one of them is poisoned.

The poison exhibits no symptoms until death. Death occurs inevitably within ten to twenty hours after consuming even the minutest amount of poison. You have over a thousand slaves at your disposal and less than 24 hours to determine which single barrel is poisoned. You also have thirty prisoners awaiting execution. Both prisoners and slaves may be used as wine tasters, with a priority on using prisoners (as they are going to die anyway and so are considered expendable).

1. Devise a plan for identifying the poisoned barrel of wine that minimizes the number of deaths.

**(4 marks)**

1. Devise a plan for identifying the poisoned barrel of wine that utilizes all thirty prisoners awaiting execution.

**(5 marks)**

1. Devise a plan for identifying the poisoned barrel of wine that minimizes the number of wine tasters.

**(8 marks)**

For each plan:

* State how many barrels of wine each wine taster must taste.
* Explain how the barrels are numbered.
* Explain which of the numbered barrel(s) each wine taster must test.
* State the number of deaths due to poison.

**(Total 25 Marks)**

**Q. 5**

**(a)**

Explain, with the aid of a diagram, the concept of public key cryptography (asymmetric key cryptography). Include two advantages and two disadvantages of the method as part of your explanation.

**(9 marks)**

**(b)**

A secret agent wants to send you the message “OK” encoded using the RSA algorithm. Suppose *n* = *p*×*q* = 17×59 = 1,003 and the encryption key *e* = 3.

1. Convert the message to plaintext and then to ciphertext.

**(4 marks)**

1. You receive the ciphertext number and must decipher it. Find the value of the decryption key *d* and explain how the ciphertext number can then be decrypted.

**(6 marks)**

**(c)**

1. Is 215 – 1 prime? Explain your answer.

**(2 marks)**

1. Is 220 + 1 prime? Explain your answer.

**(2 marks)**

1. Prove that the sum of two odd prime numbers is never prime.

**(2 marks)**

**(Total 25 Marks)**

1. A *talent* is a unit of weight, approximately 1 kilogram. [↑](#footnote-ref-1)